OPTIMIZATION-BASED DESIGN OF CONTROL SYSTEMS

FOR FLEXIBLE STRUCTURES

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259

ABSTRACT

The purpose of this presentation is to show that it is possible to use nonsmooth optimization algorithms to design both closed-loop finite dimensional compensators and open-loop optimal controls for flexible structures modeled by partial differential equations.

An important feature of our approach is that it does not require modal decomposition and hence is immune to instabilities caused by spillover effects. Furthermore, it can be used to design control systems for structures that are modeled by mixed systems of coupled ordinary and partial differential equations.

DESIGN OF STABILIZING FEEDBACK-SYSTEM COMPENSATORS

The optimization-based design of finite dimensional compensators for systems modeled by coupled systems of ordinary and partial differential equations is made possible by a generalization of the following necessary and sufficient stability test for linear systems described by ordinary differential equations.

THE DYNAMICAL SYSTEM

Consider a parametrized, linear, time-invariant, interconnected, finite dimensional dynamical system, $\Sigma(p)$, described by a set of state equations:

$$\dot{x}(t) = A(p)x_i(t) + B(p)u(t),$$
$$y(t) = C(p)x(t) + D(p)u(t), \tag{1}$$

We shall denote the characteristic polynomial of $\Sigma(p)$ by $\chi(s,p)$ and assume that the coefficients of $\chi(s,p)$ are continuously differentiable in p.

261

S-STABILITY

When, it is desired to ensure not only exponential stability of a closed loop system, but also to exercise some control over the location of its poles, it is convenient to make use of the following definition of S-stability.

Definition *(S-stability):* Consider a linear, time-invariant, finite dimensional dynamical system Σ of the form (1). Let **S** be an open unbounded subset of $\mathbb{C}$ which is symmetrical with respect to the real axis, and such that $\mathbf{S^c} \supset \mathbf{C_+}$, where $\mathbf{S^c}$ is the complement of **S** and $\mathbf{C_+}$ is the closed right half of the complex plane.

We say that the system Σ is **S–stable** if all the zeros of its characteristic polynomial are in **S**. ■

A MODIFIED NYQUIST STABILITY CRITERION

Theorem :   Let $S \subset \mathbb{C}$ be as specified in the Definition and let $B \subset \mathbb{C}$ be any simply connected set satisfying $(0,0) \notin B$. Suppose that $D(s,q) \in \mathbb{C}[s]$ is a parametrized polynomial of degree N, whose coefficients depend on the parameter vector $q \in \mathbb{R}^{n_D}$ in such a way that for every $\chi(s) \in P_N$ satisfying $Z[\chi(s)] \subset S$, there exists a $q_\chi \in \mathbb{R}^{n_D}$ such that

(i) $Z[D(s,q_\chi)] \subset S$,   (2a)

(ii) $\chi(s)/D(s,q_\chi) \in B$, $\forall\ s \in \partial S$.   (2b)

Then, given a polynomial $\chi(s) \in P_N$, $Z[\chi(s)] \subset S$ if and only if there exists a $q_\chi \in \mathbb{R}^{n_D}$ such that (2a,b) hold.   ∎

PROOF OF MODIFIED NYQUIST STABILITY CRITERION

(= = =>)   Suppose that $Z[\chi(s)] \subset S$. Then, by assumption, there exists a $q_\chi \in \mathbb{R}^{n_D}$ such that (2a), (2b) hold.

(<= = =)   Next, suppose that (2a), (2b) hold. Then, because B is a simply connected set which does not contain the origin, the locus traced out in the complex plane by $\chi(s)/D(s,q_\chi)$, for $s \in \partial S$, does not encircle the origin. It now follows from (2a) and the Argument Principle that $Z[\chi(s)] \subset S$.   ∎

Comment :   It is clear from the Theorem that an acceptable parametrization of the polynomial $D(s,q)$ depends on the shape of the set S and the choice of the set B. A further requirement is imposed by semi-infinite optimization: the parametrization must be such that it is easy to ensure that the zeros of $D(s,q)$ are in S.   ∎

OPTIMIZATION-BASED CONTROL SYSTEM DESIGN
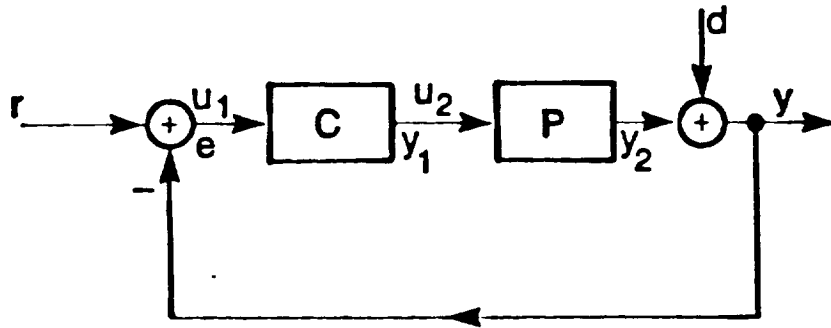
SYSTEM DYNAMICS

$$\frac{d}{dt}\begin{bmatrix} z_P^1 \\ z_P^2 \\ z_P^3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & -2 \\ 1 & 0 & 0 \\ 0 & -2 & -4 \end{bmatrix}\begin{bmatrix} z_P^1 \\ z_P^2 \\ z_P^3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} u_2^1 \\ u_2^2 \end{bmatrix},$$

$$\begin{bmatrix} y_2^1 \\ y_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} z_P^1 \\ z_P^2 \\ z_P^3 \end{bmatrix}.$$

$$\frac{d}{dt}\begin{bmatrix} z_C^1 \\ z_C^2 \end{bmatrix} = \begin{bmatrix} x^1 & x^2 \\ x^3 & x^4 \end{bmatrix}\begin{bmatrix} u_1^1 \\ u_1^2 \end{bmatrix}, \qquad \begin{bmatrix} y_1^1 \\ y_1^2 \end{bmatrix} = \begin{bmatrix} x^5 & x^6 \\ x^7 & x^8 \end{bmatrix}\begin{bmatrix} z_C^1 \\ z_C^2 \end{bmatrix}.$$

DESIGN VECTOR: $x = [\, x^1, x^2, \ldots, x^8\,]$.

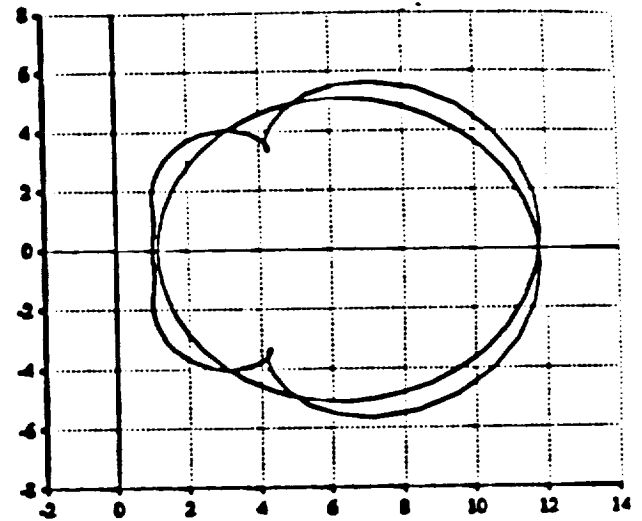# DESIGN CRITERIA

1. The feedback system must be exponentially stable.

2. The system should have a good step input response.

3. There should be little interaction between channels.

4. Plant should not be saturated by command input effects.

5. System should have high output disturbance rejection.
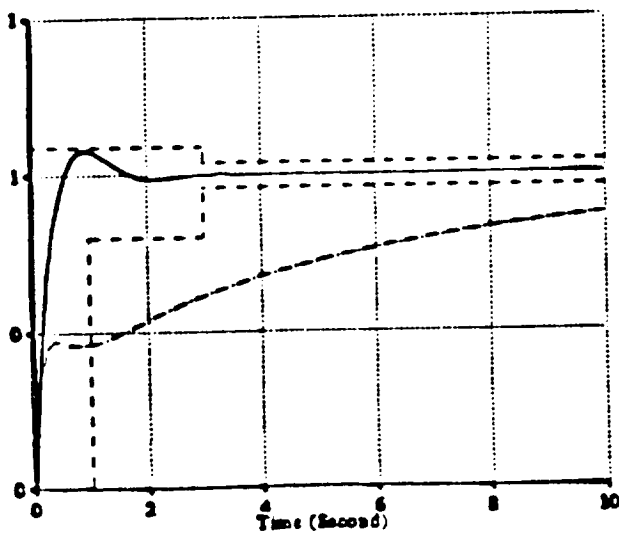
266
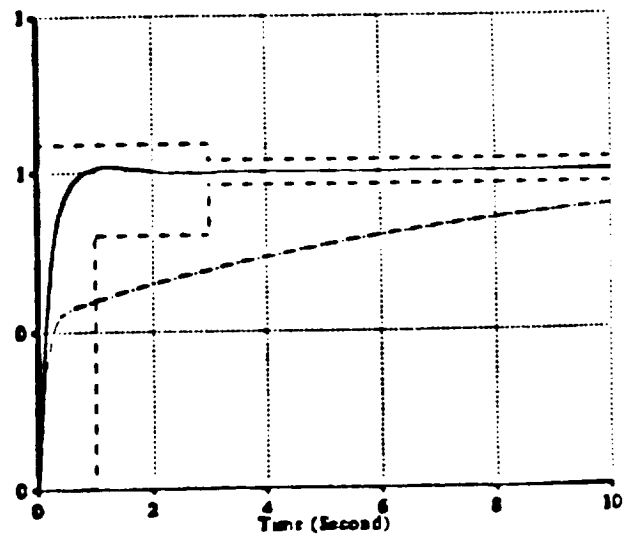
MODIFIED NYQUIST STABILITY CONSTRAINT
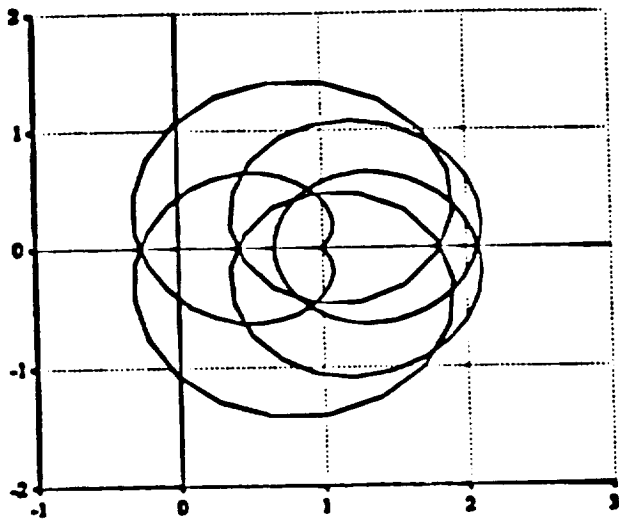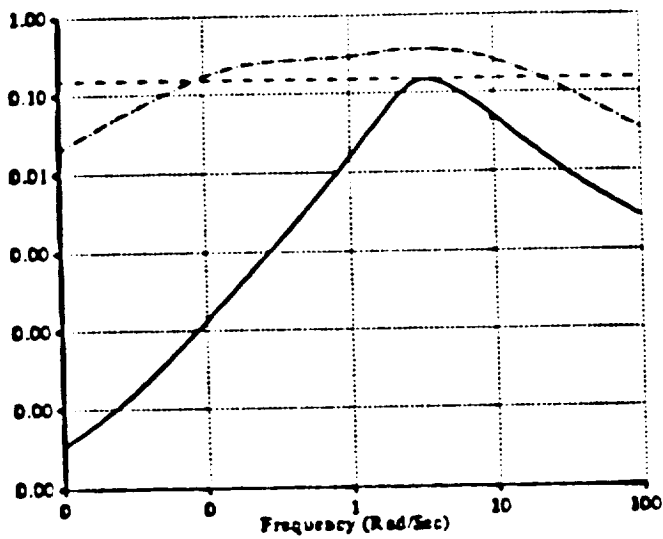

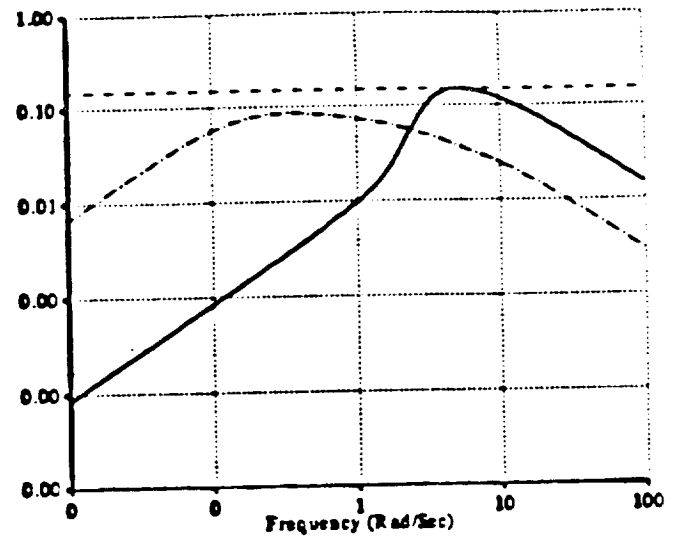Initial Design

Final Design

STEP RESPONSES

Channel 1
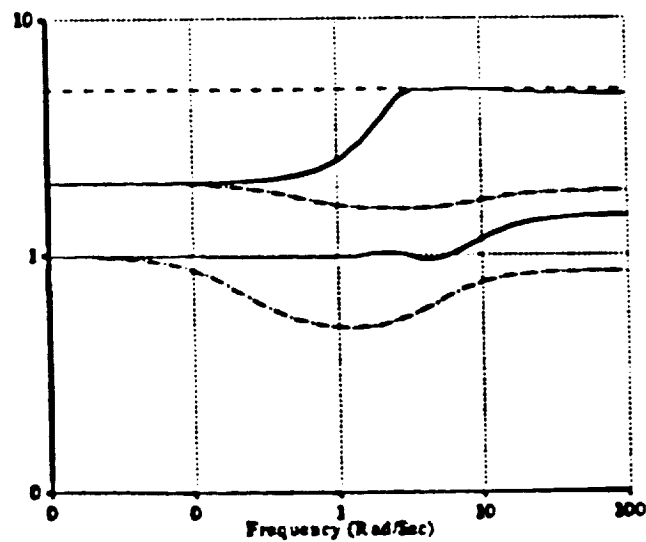
Channel 2

267

CHANNEL INTERACTION CONSTRAINT

Magnitude of $H_{y^2 r^1}(j\omega, x)$

Magnitude of $H_{y^1 r^2}(j\omega, x)$

COMMAND INPUT SATURATION CONSTRAINT

Singular Values of $H_{u_2 r}(j\omega, x)$
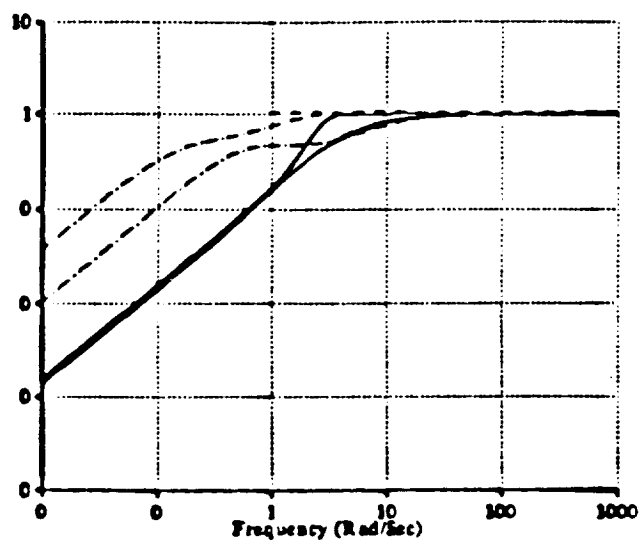
OUTPUT DISTURBANCE SUPPRESSION CONSTRAINT

Must accept some disturbance amplification outside operating bandwidth:

$$\bar{\sigma}[H_{yd}(j\omega, x) \le 1.05, \ \forall \ \omega \in [1, 1000]$$
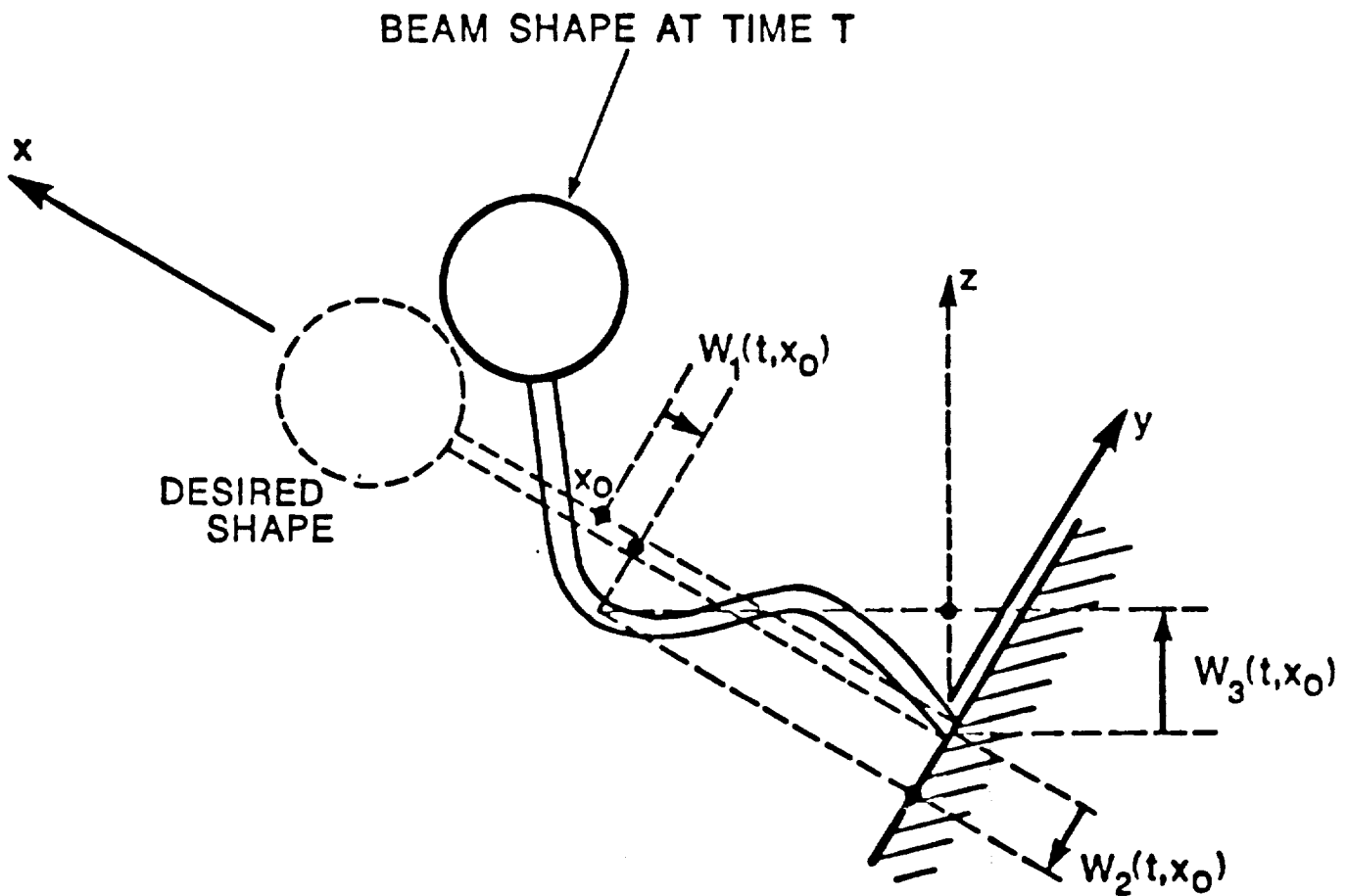
COST: OUTPUT DISTURBANCE SUPPRESSION

Suppress disturbance effects inside operating bandwidth:

$$f(x) \triangleq \max_{\omega \in [0.001, 1]} \bar{\sigma}[H_{yd}(j\omega, x)$$

Singular Values of $H_{yd}(j\omega, x)$

269

INTEGRATED STRUCTURE-CONTROL-SYSTEM DESIGN

Vibrating Beam

270

DYNAMICS

- **GENERAL MODEL:** Euler-Bernoulli Model, Kelvin-Voigt or Proportional Damping, Coupled Axial and Flexural Linear PDE's.

- Control Forces $F^i(t)$, Actuator Positions a^i, Sensor Positions s^i.

- **SIMPLIFIED MODEL:** Decoupled Motion Formulation:

$$mu_{tt}(t\ ,\ x) + CIu_{txxxx} + EIu_{xxxx}(t\ ,\ x) = \sum_{i=1}^{n_i} b_i(x - a_i)F^i(t)\ .$$

$$\dot{y}^i(t) = \int_0^1 c_i(\zeta - s^i)u(t\ ,\ \zeta)d\zeta \quad \text{or} \quad y^i(t) = \int_0^1 d_i(\zeta - s^i)\dot{u}(t\ ,\ \zeta)d\zeta\ .$$

BOUNDARY CONDITIONS

$$u(t\ ,\ 0) = 0\ ,\quad u_x(t\ ,\ 0) = 0,\ Ju_{ttx}(t\ ,\ 1) + CIu_{txx}(t\ ,\ 1) + EIu_{xx}(t\ ,\ 1) = 0\ ,$$

$$Mu_{tt}(t\ ,\ 1) - CIu_{txxx}(t\ ,\ 1) - EIu_{xxx}(t\ ,\ 1) = 0.$$

271

DESIGN CRITERIA

1. The feedback system must be exponentially stable.

2. Control system compensator should be finite dimensional.

3. Actuators should not be saturated by command input effects.

4. System should have high mechanical disturbance rejection.

5. Average power use should be low.

6. Structure weight should be low.

7. Structure should remain in elastic range.

272

DESIGN VARIABLES

- ## CONTROL SYSTEM COMPENSATOR

(i) Coefficients of compensator differential equation.

- ## STRUCTURE

(i) Positions of actuators and sensors.

(ii) Parameters of damping devices.

(iii) Parameters of composite materials.

(iv) Parameters determining shape of structure.

PRELIMINARY RESULTS

1. The control system can be stabilized using a *finite dimensional* proportional-plus-integral controller which ensures good disturbance rejection. The use of our modified Nyquist stability criterion in the design of a stabilizing controller requires only evaluations of the system frequency response. Since the frequency response at a given frequency can be computed in some cases by formula and in the more general cases by solving two-point linear boundary value problems, *there is no need for modal decomposition* and hence *there are no spillover effects.* As in the finite dimensional case, time and frequency domain constraints can be treated simultaneously and, in an *integrated design approach* structural parameters and constraints can also be introduced into the optimization problem.

2. If *a sequential design approach* is used, an infinite dimensional compensator can be designed using an H^∞ frequency domain constraint formulation which results in a convex optimization problem and automatically ensures exponential stability with stability margin.

3. An infinite dimensional controller designed as above can be approximated by a finite dimensional controller without spillover effects.

4. A special semi-infinite optimization algorithm has been developed which is highly effective for design with H^∞ frequency domain design constraints.

274

A FLEXIBLE ARM OPTIMAL SLEWING PROBLEM

$\theta = 45°$

$\omega(t,x)$

275
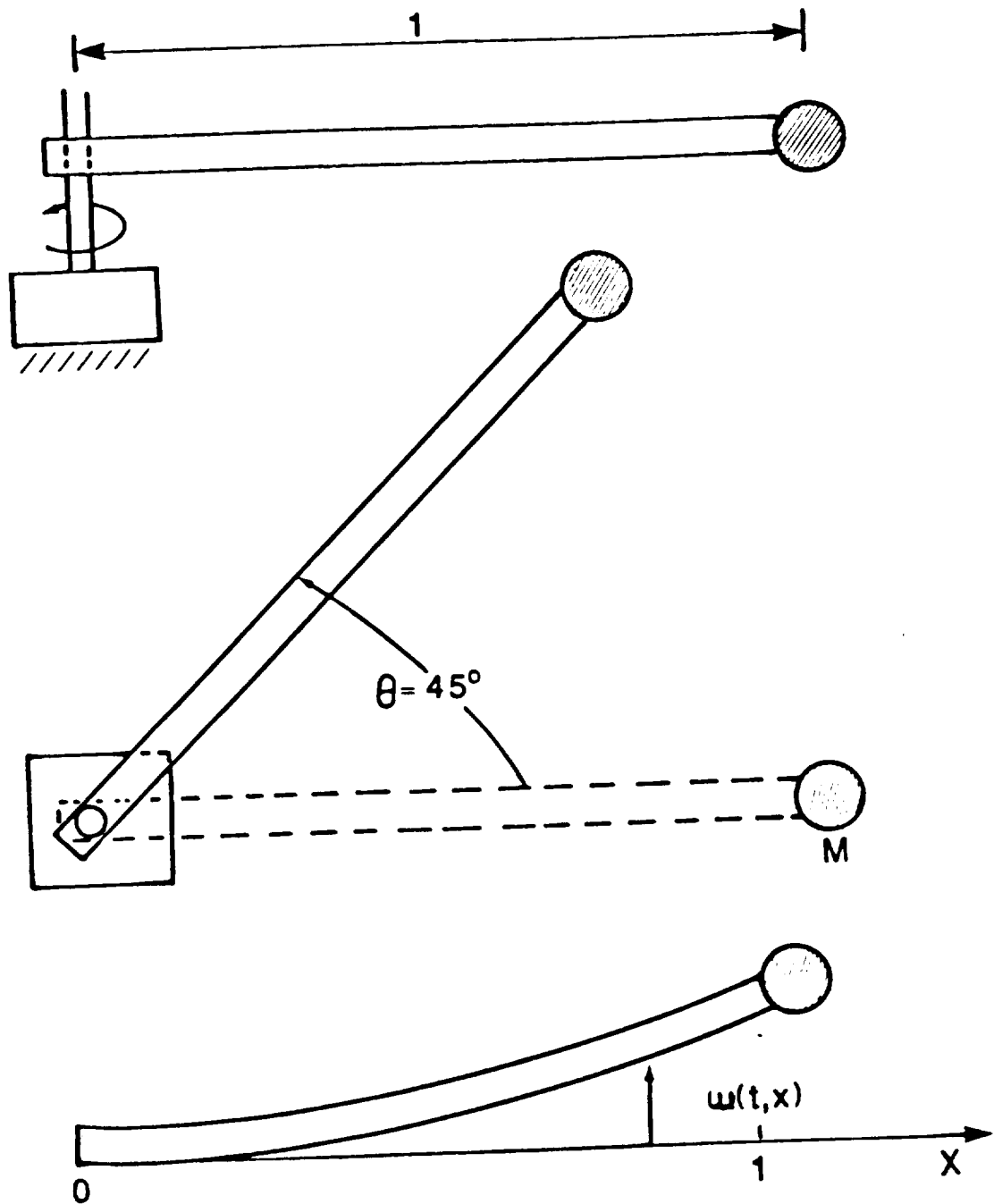
THE DYNAMICAL SYSTEM

Hollow aluminum tube: one meter long, 2.0 cm diameter, 1.6 mm thick. Attached mass weighs 1 kg. We assume that motor torque u(t) can be directly controlled.

Standard Euler-Bernoulli tube equations with Kelvin-Voigt visco-elastic damping:

$$mw_{tt}(t,x) + CIw_{txxxx}(t,x) + EIw_{xxxx}(t,x) - m\Omega^2(t)w(t,x)$$
$$= - mu(t)x, \quad x \in [0,1] \tag{1a}$$

with boundary conditions:

$$w(t,0) = 0, \quad w_x(t,0) = 0, \quad CIw_{txx}(t,1) + EIw_{xx}(t,1) = 0. \tag{1b}$$

$$M(\Omega^2(t)w(t,1) - w_{tt}(t,1) - u(t)) + CIw_{txxx}(t,1) + EIw_{xxx}(t,1) = 0 \tag{1c}$$

where w(t,x) is displacement of tube from *shadow tube* (which remains undeformed during the motion), u(t) is motor torque, and $\Omega(t)$ rad/sed is angular velocity. Above: m = .2815 kg/m, C = 6.89×10^7 pascals/sec., E = 6.89×10^9 pascals, I = 1.005 × 10^{-8}m^4, The tube is very lightly damped (0.1 per cent).

276

THREE OPTIMAL SLEWING PROBLEMS

P$_1$:

Minimize the time required to rotate the tube 45 degrees, from rest to rest, subject to the torque not exceeding 5 newton-meters.

P$_2$:

Minimize the total energy required to rotate the tube 45 degrees, from rest to rest, subject to the torque not exceeding 5 newton-meters and the maneuver time not exceeding a given bound.

P$_3$:

Minimize the time required to rotate the tube 45 degrees, from rest to rest, subject to the torque not exceeding 5 newton-meters and an upper bound on the potential energy due to deformation of the tube throughout the entire maneuver.

THE DYNAMICAL SYSTEM

MATHEMATICAL FORMULATION OF THE THREE PROBLEMS

- To avoid technical problems associated with variable intervals and problems due to discretization, augment dynamics by one state variable and introduce scale factor $T > 0$ so that problem becomes defined on *normalized time* interval $[0,1]$, with T also equal to final time.

- *Tube is at rest* when the total energy = energy due to rigid body motion + energy due to vibration and deformation = 0.

(i) To ensure a slewing motion of 45°, we define

$$g^1(u,T) \triangleq (\Theta - \Pi/4)^2 \qquad (2)$$

(ii) Rigid body energy at final time is proportional to the square of the angular velocity.

$$g^2(u,T) \triangleq \Omega(T)^2. \qquad (3)$$

278

(iii) Kinetic energy due to tube vibration at normalized time 1 is

$$g^3(t, u) \triangleq \frac{m}{2} \int_0^1 w_t(1, x)^2 dx. \tag{4}$$

(iv) Potential energy due to tube deformation at normalized time 1 is

$$g^4(1, u) \triangleq \frac{EI}{2} \int_0^1 w_{xx}(1, x)^2 dx. \tag{5}$$

- Potential energy due to deformation of the tube at normalized time t:

$$P(t, u) \triangleq \frac{EI}{2} \int_0^1 w_{xx}(t, x)^2 dx. \tag{6}$$

(v) To limit tube deformation for all $t \in [0, 1]$ we define

$$g^5(u, T) \triangleq \int_0^T [\max\{ P(t, u) - f(t), 0 \}]^2 \tag{7}$$

(vi) To ensure slewing time does not exceed T_f seconds, we define

$$g^6(u, T) \triangleq T - T_f. \tag{8}$$

279

FINAL PROBLEM FORM

P_1: $\displaystyle\min_{T \in \mathbb{R}_+, \, u \in G}$ $\{ \, g^0(u,T) \mid g^j(u,T) - \varepsilon \leq 0, \, j \in \{1,2,3,4\} \, \}$,

where $g^0(u,T) \triangleq T$, $\mathbb{R}_+ \triangleq \{ \, \gamma \in \mathbb{R} \mid \gamma > 0 \, \}$ and

$G \triangleq \{ \, u \in L_\infty[0,1] \mid |u(t)| \leq 5, \, t \in [0,1] \, \}$.

P_2: $\displaystyle\min_{T \in \mathbb{R}_+, \, u \in G}$ $\{ \, g^0(u,T) \mid g^j(u,T) - \varepsilon \leq 0, \, j \in \{1,2,3,4,6\} \, \}$,

where $g^0(u,T) \triangleq \displaystyle\int_0^1 \|u(t)\|^2 \, dt$.

P_3: $\displaystyle\min_{T \in \mathbb{R}_+, \, u \in G}$ $\{ \, g^0(u,T) \mid g^j(u,T) - \varepsilon \leq 0, \, j \in \{1,2,3,4,5\} \, \}$,

where $g^0(u,T) \triangleq T$.
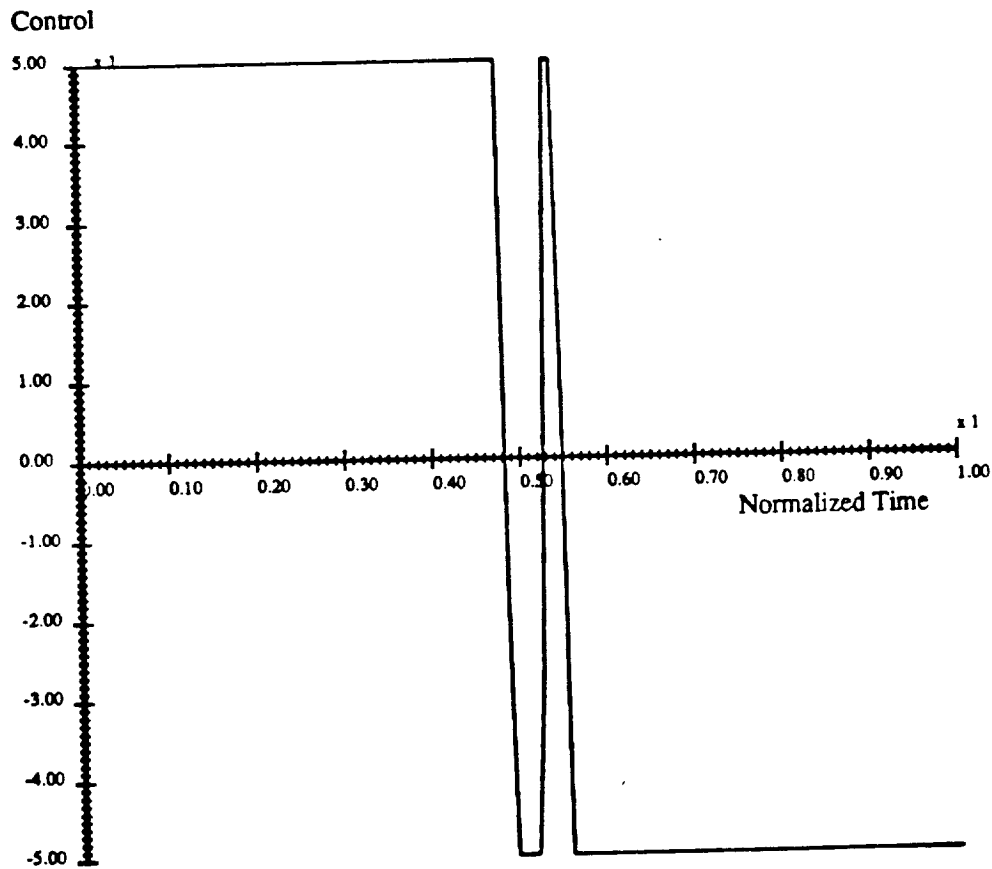
- All g^j are continuously differentiable in $L_\infty[0,1]$.
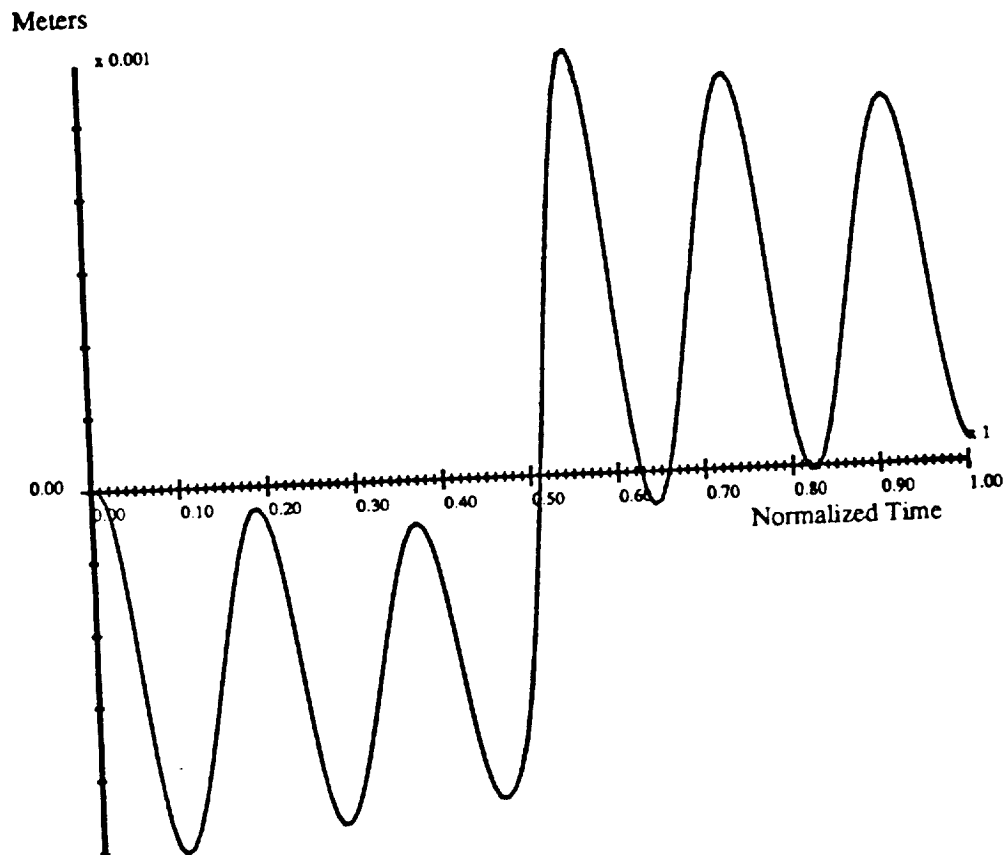
THE DYNAMICAL SYSTEM

COMPUTATIONAL RESULTS

IMPLEMENTATION. Because we cannot solve the system PDEs exactly, we cannot evaluate $g^j(u,T)$ or $\nabla g^j(u,T)$ exactly. Furthermore, since u is an infinite dimensional design vector, it can only be entered into a computer in discretized form. We use an *implementable* algorithm which adjusts integration precision and control discretization adaptively. To discretize the PDE in space, we use the finite element method. Since the PDE is fourth order in space, it is necessary to use elements of at least second order. We have chosen Hermite splines as basis elements. The input $u \in G$ is discretized in time and Newmark's method is applied to evaluate the resulting system of ordinary differential equations.

LINEARIZATION. The results presented are for the case in which the $\Omega^2(t)$ terms are neglected in equation (1). Similar results have been obtained by performing experiments when the $\Omega^2(t)$ terms are included.

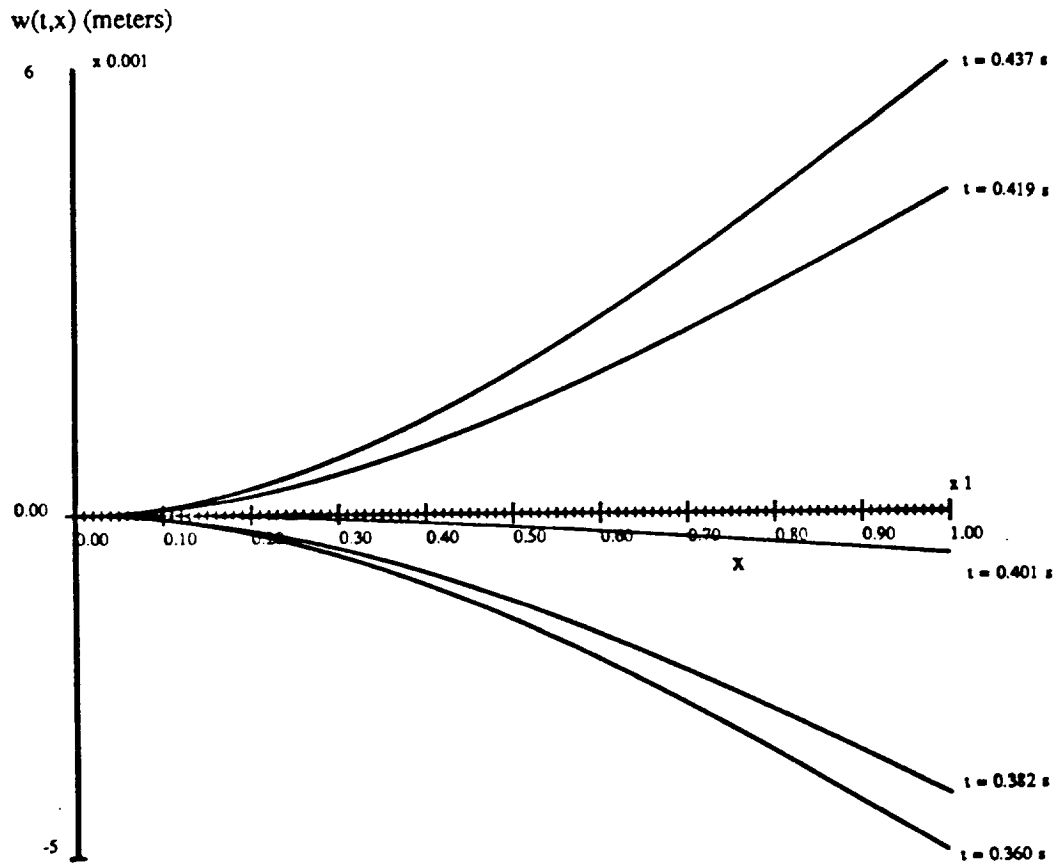# OPTIMAL CONTROL FOR MINIMUM-TIME PROBLEM

WITH TORQUE CONSTRAINTS ONLY

282

C 4

TIP DISPLACEMENT FOR MINIMUM-TIME PROBLEM

WITH TORQUE CONSTRAINTS ONLY

Meters

x 0.001

Normalized Time

0.00   0.10   0.20   0.30   0.40   0.50   0.60   0.70   0.80   0.90   1.00
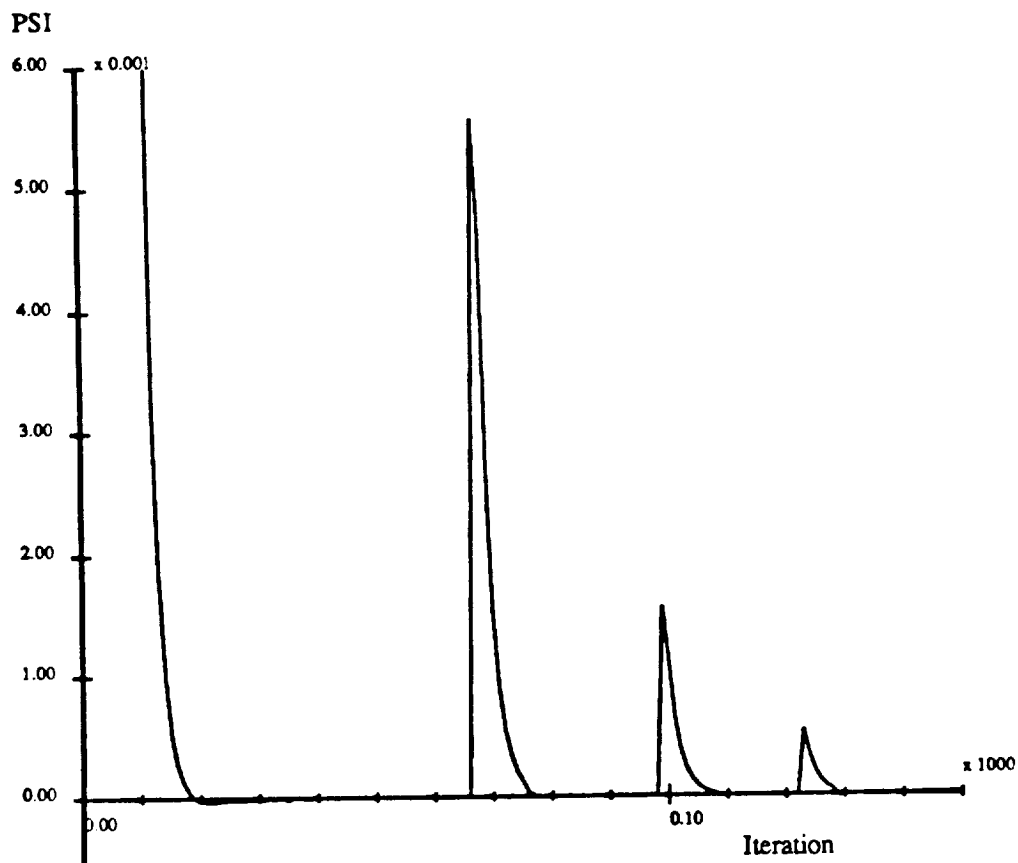
283

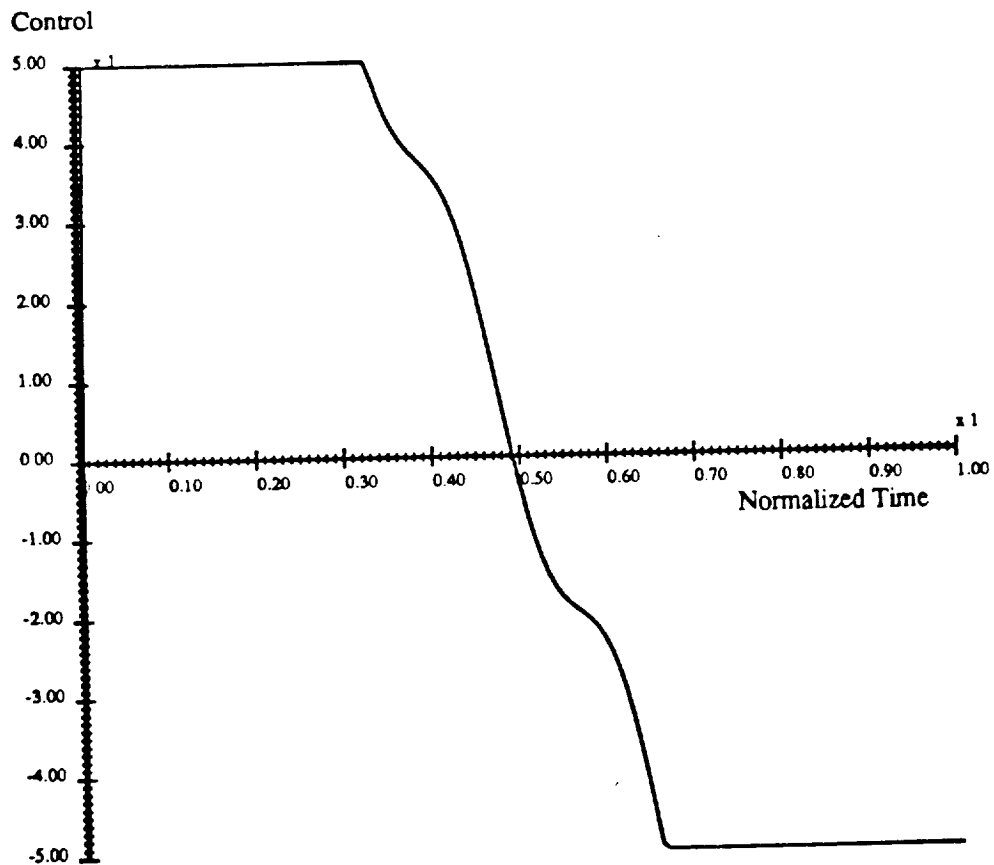# DEVIATION FROM SHADOW BEAM FOR MINIMUM-TIME

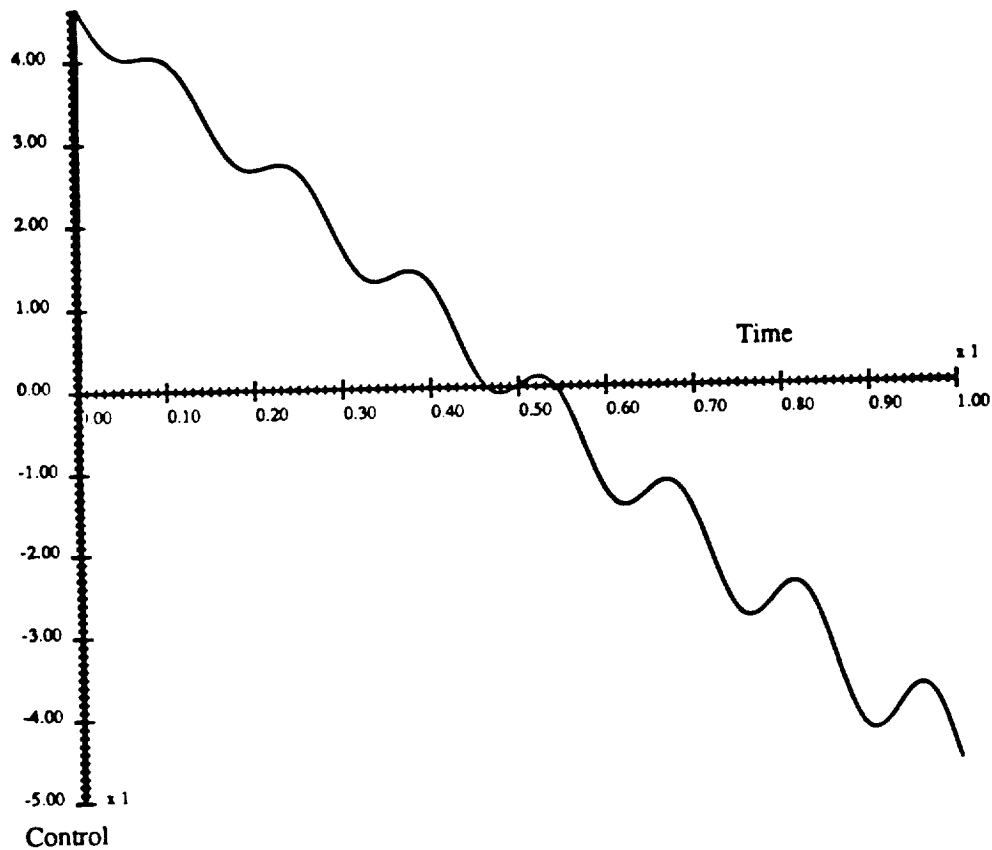# PROBLEM WITH TORQUE CONSTRAINTS ONLY

284

CONSTRAINT VIOLATION FOR MINIMUM-TIME PROBLEM

WITH TORQUE CONSTRAINTS ONLY:

DISCRETIZATION EFFECTS

285

OPTIMAL TORQUE

FOR MINIMUM-CONTROL-ENERGY PROBLEM

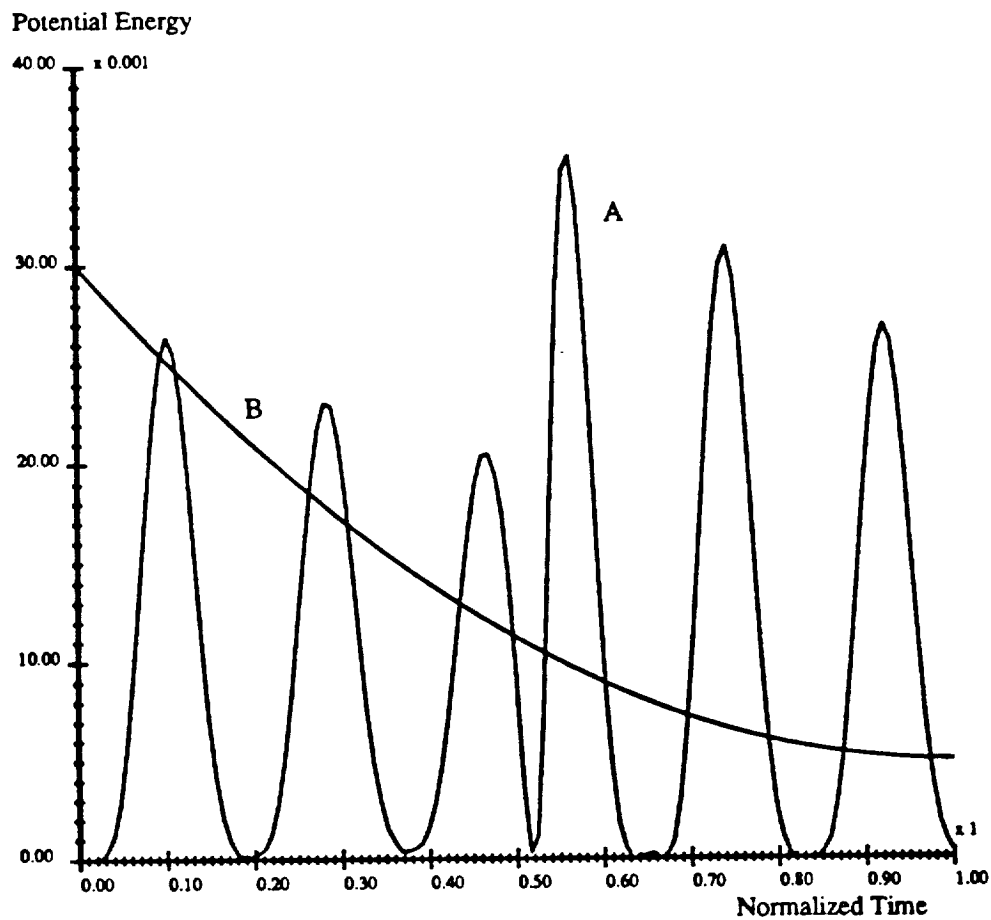# WITH TORQUE CONSTRAINTS AND FINAL TIME ≤ 0.8 SEC.

286

OPTIMAL TORQUE

FOR MINIMUM-CONTROL-ENERGY PROBLEM

WITH TORQUE CONSTRAINTS AND FINAL TIME $\leq$ 1.0 SEC.

287

POTENTIAL ENERGY FOR MINIMUM TIME PROBLEM

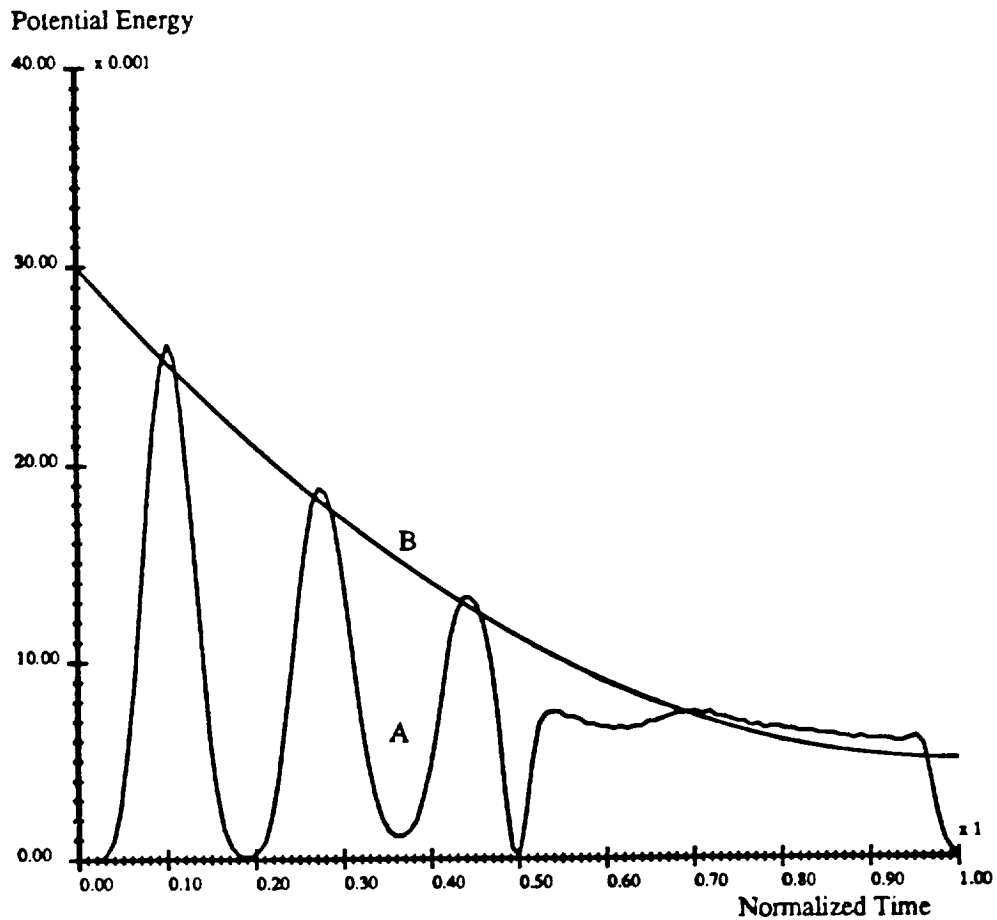# WITH TORQUE CONSTRAINTS ONLY

Curve A is potential energy

Parabola B is deformation constraint.

288

POTENTIAL ENERGY FOR MINIMUM-TIME PROBLEM

WITH TORQUE AND POTENTIAL ENERGY CONSTRAINTS

Curve A is potential energy

Parabola B is deformation constraint.

Potential Energy

OPTIMAL CONTROL FOR MINIMUM-TIME PROBLEM

WITH TORQUE AND POTENTIAL ENERGY CONSTRAINTS

Note: The optimal final time is 0.8177 seconds, an increase of only 3.7 percent over the solution of P_1.

290